

RADIATIVE SYMMETRY BREAKING IN THE
SUPERSYMMETRIC MINIMAL $B - L$
EXTENDED STANDARD MODEL

by

ZACHARY BURELL

NOBUCHIKA OKADA, COMMITTEE CHAIR

LOUIS J. CLAVELLI
CONOR HENDERSON
GREG SZULCZEWSKI

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ABSTRACT

The Standard Model (SM) of particle physics is a precise model of electroweak interactions, however there is growing tension between the SM and observations (neutrino oscillations, dark matter, dark energy, baryogenesis, among others). There is no reason to expect the validity of the ad hoc SM to remain intact at energy scales above a few TeV, thus a more fundamental theory will almost certainly be required.

Motivated by these considerations, we investigate a Supersymmetric version of a natural extension of the SM, the $U(1)_{B-L}$ model, that is obtained by gauging the accidental B-L symmetry that exists in the ordinary SM. The Supersymmetric $U(1)_{B-L}$ extended SM can resolve the neutrino mass problem, the dark matter problem, the hierarchy problem, and provides a mechanism for establishing the observed baryon asymmetry of the Universe.

When we include quantum corrections to the Higgs potential of the model, we find that Radiative $B - L$ symmetry breaking occurs through the interplay between large Majorana Yukawa couplings and SUSY breaking masses, and the SM neutrino masses arise through the Type-I seesaw mechanism. We deduce from the form RGEs that $B - L$ will be broken near the TeV scale when the Majorana Yukawa couplings are of order 0.5, leading to TeV scale right-handed neutrinos, which could be accessible at the LHC. We discuss the different vacua of the theory and the viability of a right-handed neutrino dark matter candidate in the R-parity conserving and violating sectors. A new \mathbb{Z}_2 parity is postulated in the R-parity violating sector in which case the lightest right handed neutrino becomes a viable Dark Matter candidate in addition to the gravitino.

DEDICATION

My thesis is dedicated to my family whose love and support made writing this thesis a possibility.

LIST OF ABBREVIATIONS AND SYMBOLS

SM	the Standard Model
GeV	10^9 electron-volts
TeV	10^{12} electron-volts
EWSB	Electroweak symmetry breaking
VEV	Vacuum expectation value
GUT	Grand unified theory
SSB	Spontaneous symmetry breaking
SUSY	Supersymmetry/Supersymmetric
MSSM	Minimal Supersymmetric Standard Model
LHC	Large Hadron Collider
mSUGRA	Minimal Supergravity
FCNC	Flavor changing neutral currents
x^μ	Space time coordinates ($\mu = 0, 1, 2, 3$)
$g^{\mu\nu}$	Metric tensor diag $(1, -1, -1, -1)$
g_s	Strong coupling constant
g	Weak coupling constant
g'	$U(1)_Y$ coupling constant
g_{BL}	The $U(1)_{B-L}$ coupling constant.
\mathcal{L}	Lagrangian density
Φ	Left handed chiral superfield
ϕ	Left handed scalar component of chiral superfield
$\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$	Anti-commuting grassmanian coordinates of chiral superspace ($\alpha, \dot{\alpha} = 1, 2$)
ν_i^c	Right handed neutrinos($i = 1, 2, 3$)
e^c	Right handed positrons
Y	Yukawa coupling

H_u	MSSM “up” type Higgs left handed chiral superfield
H_d	MSSM “down” type Higgs left handed chiral superfield
\mathcal{W}	Super-potential
\mathcal{K}	Kahler-potential
M_{GUT}	Grand unification scale ($M_{GUT} \sim 10^{16}$ GeV)
M_{PL}	Planck scale-the fundamental scale of nature ($M_{PL} \sim 10^{19}$ GeV)
Λ_S	Supersymmetry breaking scale
B	Baryon number
L	Lepton number
$B - L$	Baryon number - Lepton number

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Chapter 1

The Standard Electroweak Model

1.1 Introduction

The Standard Model is certainly a triumphant stepping stone in the progression to elucidate nature's fundamental properties; it accurately describes the fundamental particles, the quarks and leptons, and their interactions, with the exception of gravity, down to distance scales $d \sim 10^{-16}$ cm [1, 2].

The SM is not flawless however, and many observations cannot be explained by it alone. These include solar and atmospheric neutrino oscillations [3, 4, 5, 6, 7, 8], the existence of non-baryonic dark matter [9, 10, 11, 12], the observed baryon asymmetry of the Universe [13], the Muon anomalous magnetic moment [14], and the recent anomalous like sign di-muon charge asymmetry measured by the DØ experiment [15]. Moreover, there are a number of different theoretical suggestions for the incompleteness of the SM, the most pressing among these is the gauge hierarchy problem, which we discuss in more detail in section 1.2.

Neutrino flavor oscillations cannot be understood without an extension of the SM, because the neutrino is massless in the standard framework. However, the phenomenon of neutrino oscillations arises only in the case of distinct non-zero neutrino masses between different neutrino flavors. The unambiguous observation of neutrino oscillations has presented a challenge to theorists for decades and formally speaking, it remains a challenge. However, the introduction of one or more additional heavy right handed neutrinos into the model gives rise to the see-saw mechanism, which can explain the smallness of the neutrino masses under certain additional requirements.

The existence of non-baryonic dark matter is another clear indication that an extension of the SM is required. Dark matter is not composed of quarks and leptons, and thus does not fall within the framework of ordinary matter, i.e. the SM. Evidence for dark matter has been accumulating now for about three quarters of a century, beginning with Fritz Zwicky's observation in 1933 that galaxies in the coma cluster were spinning much faster than could be accounted for by the observed amount of normal matter they contained [16]. The measurements of individual galaxy rotation curves performed by Vera Rubin and Albert Bosma in 1970's, confirmed Zwicky's observations, also suggesting the existence of dark matter [17, 18]. These classical observations have been confirmed and extended by more recent, higher precision observations. Data from weak [19], and strong [20] gravitational lensing, the Bullet Cluster [12], Big Bang Nucleosynthesis [21], Large scale structure [22], distant supernovae [23], and the cosmic microwave background (CMB) radiation [24], are consistent and all suggest that Dark Matter exists in five times the abundance of the ordinary matter explained by the SM. The incompleteness of the SM is evident from its inability to account for these observations without extension or modification.

1.2 The Standard Model

The Standard Model is a gauge field theory of three generations of matter based on the gauge group $\mathcal{G}_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, each with its own running coupling constant, g_s, g, g' . We first present the bare SM formalism, and then introduce spontaneous symmetry breaking (SSB). We conclude with the massive SM Lagrangian after SSB.

For the SM, the Lagrangian is

$$\mathcal{L}_{SM} = \mathcal{L}_{SU(3)} + \mathcal{L}_{SU(2) \otimes U(1)} \quad (1)$$

where $\mathcal{L}_{SU(3)}$ is the perturbative QCD sector Lagrangian and $\mathcal{L}_{SU(2) \otimes U(1)}$ is the Lagrangian

for the electroweak sector.

1.3 The QCD sector

The strong interaction arises from color charge. The fermions which carry color quantum numbers are the quarks. There is an associated set of massless gauge bosons, the *gluons*, which also carry color and mediate the strong-color interaction, the theory which elucidates this phenomenology is Quantum Chromodynamics or QCD. Yukawa was among the first to consider “strong” interactions during the 1930’s, motivated by the newly discovered pion mediated nucleon-nucleon interactions being observed at that time. Finally in 1964 Murray Gell-Mann [25] and Georg Zweig [26] introduced the eight-fold way phenomenology of color mediated interactions.

The perturbative QCD Lagrangian is

$$\mathcal{L}_{SU(3)} = -\frac{1}{4}\mathcal{F}_{\mu\nu}^i\mathcal{F}^{i\mu\nu} + \sum_r \bar{Q}_{r\alpha} i \not{D}_\beta^\alpha Q_r^\beta \quad (2)$$

where we sum over quark flavor $r \in [\text{up, down, charm, strange, bottom, top}]$ and $\alpha, \beta = 1, 2, 3$ are color indices. The non-abelian field strength tensor for the gluon fields \mathcal{G}_μ^i is, (for $i, j, k = 1, \dots, 8$) :

$$\mathcal{F}_{\mu\nu}^i = \partial_\mu \mathcal{G}_\nu^i - \partial_\nu \mathcal{G}_\mu^i - g_s f_{ijk} \mathcal{G}_\mu^j \mathcal{G}_\nu^k \quad (3)$$

where g_s is the QCD gauge coupling constant. The \mathcal{F}^2 term leads to three and four point gluon self interactions.

The structure constants $f_{ijk}(i, j, k = 1, \dots, 8)$ are defined by $[\lambda^i, \lambda^j] = 2if_{ijk}\lambda^k$, where the λ^i are the Gell-Mann matrices, shown in table 1 below.

$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$	$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$	$\lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$	

Table 1. The Gell Mann Matrices

The second term in $\mathcal{L}_{SU(3)}$ is the gauge covariant derivative for $SU(3)_c$:

$$\mathcal{D}_{\mu\beta}^\alpha = (\mathcal{D}_\mu)_{\alpha\beta} = \partial_\mu \delta_{\alpha\beta} + ig_s \mathcal{G}_\mu^i L_{\alpha\beta}^i \quad (4)$$

where the quarks transform according to the triplet representation matrices $L^i = \lambda^i/2$. The Lagrangian $\mathcal{L}_{SU(3)}$ is invariant under the $SU(3)$ gauge transformations.

The purely vector, hence parity conserving QCD color interactions are flavor diagonal but in general can mix quark colors. Also, it is clear from equation 2 that there are no bare mass terms for the quarks in the Lagrangian. Bare mass terms would be possible in QCD alone but the electroweak sector of the unified model has global chiral symmetry in the unbroken electroweak phase which forbids bare mass terms. The quark masses come in through spontaneous symmetry breaking. There are additional, effective ghost and gauge-fixing terms which enter into the quantization of both $SU(3)_C$ and $SU(2)_L \otimes U(1)_Y$ sectors of the theory. The QCD formalism is very extended and we will not be using it in what follows, so we omit these terms in this overview of the SM.

1.4 The Electroweak Sector

The Electroweak Lagrangian is

$$\mathcal{L}_{SU(2)\otimes U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_\varphi + \mathcal{L}_f + \mathcal{L}_{\text{Yukawa}} \quad (5)$$

The gauge component of the Lagrangian is:

$$\mathcal{L}_{gauge} = -\frac{1}{4}\mathcal{F}_{\mu\nu}^i\mathcal{F}^{i\mu\nu} - \frac{1}{4}\mathcal{B}_{\mu\nu}\mathcal{B}^{\mu\nu} \quad (6)$$

where the abelian $\mathcal{B}_{\mu\nu}$ and non-abelian $\mathcal{F}_{\mu\nu}^a$ field strength tensors are:

$$\mathcal{B}_{\mu\nu} = \partial_\mu\mathcal{B}_\nu - \partial_\nu\mathcal{B}_\mu \quad (7)$$

$$\mathcal{F}_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon_{abc}W_\mu^b W_\nu^c, \quad (8)$$

The superscript ' a ' runs over the adjoint representation of the gauge group. The spacetime index μ runs from 0...3, and the structure constant ϵ_{abc} is totally anti-symmetric; g is the $SU(2)$ gauge coupling, and g' is the $U(1)$ gauge coupling. The fields W_μ^a and \mathcal{B}_μ are the $SU(2)$ and $U(1)$ gauge fields, respectively. After Spontaneous Symmetry Breaking (SSB), the \mathcal{B} field mixes with W_3 to form the neutral massless photon γ , the bosonic mediator of the Electro-magnetic interaction, and the weak neutral massive Z gauge boson, the neutral massive weak mediator.

The scalar spin-zero Higgs component of the Lagrangian is:

$$\mathcal{L}_\varphi = (\mathcal{D}^\mu\varphi)^\dagger (\mathcal{D}_\mu\varphi) - V(\varphi) \quad (9)$$

where,

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \quad (10)$$

is a complex $SU(2)$ doublet with hypercharge $\mathbf{y}_\varphi = 1/2$. The gauge covariant derivative is:

$$\mathcal{D}_\mu \varphi = \left(\partial_\mu + i g \frac{\tau^i}{2} W_\mu^i + \frac{i g'}{2} \mathcal{B}_\mu \right) \varphi \quad (11)$$

where τ^i are the Pauli matrices:

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (12)$$

The scalar Higgs potential $V(\varphi)$ is given by all self interaction terms which satisfy $SU(2) \otimes U(1)$ gauge invariance. There are only two such terms, so the scalar potential is:

$$V(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \quad (13)$$

It is relevant to point out that vacuum stability requires that $\lambda > 0$, moreover, if $\mu^2 < 0$ we get SSB.

The electroweak theory is chiral, and parity is violated in the electroweak sector by assigning left handed and right handed matter fields:

$$\psi_L = (1 - \gamma_5) \frac{\psi}{2} \quad , \quad \psi_R = (1 + \gamma_5) \frac{\psi}{2} \quad (14)$$

to different representations of $SU(2) \otimes U(1)$. The fermion sector consists of $m = 3$ families

of left handed quark and lepton $SU(2)$ doublets:

$$Q_{mL} = \begin{pmatrix} U_m \\ D_m \end{pmatrix} \quad , \quad L_{mL} = \begin{pmatrix} \nu_m \\ E_m \end{pmatrix} \quad (15)$$

with corresponding right handed singlets: U_{mR} , D_{mR} , E_{mR} . The left handed ψ_L , and right handed ψ_R matter fields transform under $U(1)$ in such a way that electric charge is given by $\mathbf{q} = t^3 + \mathbf{y}$, where the t^i ($i = 1, 2, 3$) are the generators of weak isospin and \mathbf{y} is the hypercharge. The hypercharges for the doublets and singlets are given below in table 2:

	Q_L	L_L	U_R	D_R	E_R
\mathbf{y}	$+1/6$	$-1/2$	$+2/3$	$-1/3$	-1

Table 2: The SM fermion hypercharges

The fermion component of the Lagrangian is (note that the weak eigenstates Q_{mL} , L_{mL} etc. are generally mixtures of the mass eigenstates.) :

$$\mathcal{L}_f = \sum_m^F (\bar{Q}_{mL} i \not{D} Q_{mL} + \bar{L}_{mL} i \not{D} L_{mL} + \bar{U}_{mR} i \not{D} U_{mR} + \bar{D}_{mR} i \not{D} D_{mR} + \bar{E}_{mR} i \not{D} E_{mR}) \quad (16)$$

The gauge covariant derivatives for the individual matter fields are:

$$\mathcal{D}_\mu Q_{mL} = \left(\partial_\mu + i g \frac{\tau^i}{2} W_\mu^i + \frac{ig'}{6} \mathcal{B}_\mu \right) Q_{mL} \quad (17)$$

$$\mathcal{D}_\mu L_{mL} = \left(\partial_\mu + i g \frac{\tau^i}{2} W_\mu^i - \frac{ig'}{2} \mathcal{B}_\mu \right) L_{mL} \quad (18)$$

$$\mathcal{D}_\mu U_{mR} = \left(\partial_\mu + i \frac{2g'}{3} \mathcal{B}_\mu \right) U_{mR} \quad (19)$$

$$\mathcal{D}_\mu D_{mR} = \left(\partial_\mu - i \frac{g'}{3} \mathcal{B}_\mu \right) D_{mR} \quad (20)$$

$$\mathcal{D}_\mu E_{mR} = (\partial_\mu - i g' \mathcal{B}_\mu) E_{mR} \quad (21)$$

The chiral symmetry forbids any bare mass terms for the fermions. We can read off interactions between the gauge fields W_μ^i and \mathcal{B}_μ and the fermion fields $\psi_{L/R}$ from the covariant derivatives.

Finally, we must introduce a Yukawa component which couples the left handed ψ_L and right handed ψ_R matter fields. The Yukawa Lagrangian is:

$$-\mathcal{L}_{\text{Yukawa}} = \sum_{m,n=1}^F [Y_{mn}^u \bar{Q}_{mL} \varphi^c U_{nR} + Y_{mn}^d \bar{Q}_{mL} \varphi D_{nR} + Y_{mn}^e \bar{L}_{mn} \varphi E_{nR}] + \text{h.c.} \quad (22)$$

The matrices $Y_{mn}^{\{\psi\}}$ will generate mass terms after SSB, and they describe the Yukawa couplings between the single Higgs doublet φ , and the various flavors m and n of quarks and leptons. To give mass to the up quarks, down quarks, and the electron, we need Higgs representations with $\mathbf{y}_\varphi = -\frac{1}{2}$ and $\mathbf{y}_\varphi = \frac{1}{2}$. However, we do not have to introduce an additional Higgs to achieve this, since we are dealing with $SU(2)$ doublets. We only need one Higgs in this case because the conjugate representation $\bar{2}$ of the group $SU(2)$, is related to the normal representation 2 , by a similarity transformation, i.e.,

$$\varphi^c \equiv i\tau^2 \varphi^* = \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix} \quad (23)$$

which transforms as a doublet, with the required hypercharge $\mathbf{y}_{\varphi^c} = -\frac{1}{2}$.

1.5 Spontaneous Symmetry Breaking (SSB)

Spontaneous symmetry breaking occurs because the lowest energy vacuum state, represented by $|0\rangle$, does not respect gauge symmetry and thereby induces effective masses for

particles propagating through it. We therefore consider the complex vector formed by:

$$v = \langle 0 | \varphi | 0 \rangle = \text{const.} \quad (24)$$

which has components that are vacuum expectation values (VEVs) of the components of the complex scalar fields. We set $v = \text{constant}$, because any space or time dependence would increase the energy of the solution, taking us away from the minimum. Similar terms for the fermion $\langle 0 | \psi | 0 \rangle$ and boson $\langle 0 | \mathcal{A}_\mu | 0 \rangle$ fields are zero because of Lorentz invariance. We determine v by substituting $\varphi \rightarrow v = \langle 0 | \varphi | 0 \rangle$ into the scalar potential $V(\varphi \rightarrow v)$ and minimizing $V(v)$ with respect to v . The single complex Higgs doublet in the standard model can be re-written in a Hermitian basis $(\varphi_i = \varphi_i^\dagger, i = 1..4)$ as:

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 - i\varphi_2 \\ \varphi_3 - i\varphi_4 \end{pmatrix} \quad (25)$$

In the φ_i basis, the Higgs potential becomes:

$$V(\varphi) = \frac{1}{2}\mu^2 \left(\sum_{i=1}^4 \varphi_i^2 \right) + \frac{1}{4}\lambda \left(\sum_{i=1}^4 \varphi_i^2 \right)^2 \quad (26)$$

Since this is O_4 invariant, we are allowed to choose an axis in this 4d space such that $\langle 0 | \varphi_i | 0 \rangle = 0$, $i = 1, 2, 4$, and $\langle 0 | \varphi_3 | 0 \rangle = \nu$. In this basis the scalar potential transforms as:

$$V(\varphi) \rightarrow V(\nu) = \frac{1}{2}\mu^2\nu^2 + \frac{1}{4}\lambda\nu^4 \quad (27)$$

Minimizing with respect to ν , for the case when $\mu^2 < 0$, we obtain:

$$V'(\nu) = \nu(\mu^2 + \lambda\nu^2) = 0 \quad (28)$$

therefore, the minimization condition is $\nu = (-\mu^2/\lambda)^{1/2}$ and the Higgs doublet gets replaced by its classical value:

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \rightarrow v = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \quad (29)$$

We can quantize the theory by expanding around this minimum, $\varphi = \nu + \varphi'$. We can go to a basis where the Goldstone bosons disappear by performing a Kibble transformation [27] on the four Hermitian components of φ' , and employing the unitary gauge.

$$\varphi \rightarrow \varphi' = e^{-i \sum \xi^i \tau^i} \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix} \quad (30)$$

Here, H is the Higgs scalar, the ξ^i ($i = 1, 2, 3$) are three Hermitian fields, and τ^i are the Pauli matrices .

In the unitary gauge, the scalar covariant kinetic energy term takes the form:

$$(\mathcal{D}_\mu \varphi)^\dagger \mathcal{D}^\mu \varphi = \frac{1}{2} \begin{pmatrix} 0 & \nu \end{pmatrix} \left[\frac{g}{2} \tau^i W_\mu^i + \frac{g'}{2} \mathcal{B}_\mu \right]^2 \begin{pmatrix} 0 \\ \nu \end{pmatrix} + \text{higgs terms} \quad (31)$$

$$\rightarrow M_W^2 W^{+\mu} W_\mu^- + \frac{M_Z^2}{2} Z^\mu Z_\mu + \text{higgs terms} \quad (32)$$

where the kinetic energy and gauge interaction terms of the physical H particle have been omitted. Therefore, SSB gives rise to mass terms for the W and Z gauge bosons, in the form of the following mixtures of gauge eigenstates :

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W^1 \mp i W^2) \quad (33)$$

$$Z_\mu = -\sin \theta_W \mathcal{B}_\mu + \cos \theta_W W^3 \quad (34)$$

After SSB, the massless photon field \mathcal{A}_μ appears as well:

$$\mathcal{A}_\mu = \cos \theta_W \mathcal{B}_\mu + \sin \theta_W W^3 \quad (35)$$

In addition, we get mass terms for the charged weak gauge bosons:

$$M_{W^\pm} = \frac{g\nu}{2} \quad (36)$$

and the neutral weak gauge boson:

$$M_Z = \sqrt{g^2 + g'^2} \frac{\nu}{2} = \frac{M_W}{\cos \theta_W} \quad (37)$$

The Goldstone boson has disappeared from the theory but has re-emerged as the longitudinal degree of freedom of a massive vector particle. The weak angle is defined by

$$\tan \theta_W \equiv g'/g. \quad (38)$$

The weak scale is ν is given by

$$\nu = 2M_W/g \simeq \left(\sqrt{2}G_F \right)^{-1/2} \simeq 246 \text{ GeV} \quad (39)$$

Similarly, electric charge appears through the relation, $g = e/\sin \theta_W$, where e is the electric charge of the positron. Thus we have

$$M_W = M_Z \cos \theta_W \sim \frac{(\pi\alpha/\sqrt{2}G_F)^{1/2}}{\sin \theta_W} \quad (40)$$

Taking account of the fact that $\sin^2 \theta_W \sim 0.23$ and $\alpha \sim 1/129$, we arrive the leading order masses $M_Z = 91 \text{ GeV}$ and $M_W = 80 \text{ GeV}$.

After symmetry breaking the Higgs potential becomes:

$$V(\varphi) = -\frac{\mu^4}{4\lambda} - \mu^2 H^2 + \lambda \nu H^3 + \frac{\lambda}{4} H^4 \quad (41)$$

The Higgs mass term is given by:

$$M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\nu \quad (42)$$

The quartic Higgs coupling λ is unknown, so M_H is not predicted *a priori* by the SM.

The Yukawa interaction in the unitary gauge becomes

$$-\mathcal{L}_{\text{Yukawa}} = \sum_{\psi} \left(\sum_{m,n=1}^3 \bar{\psi}_{mL} Y_{mn}^{\{\psi\}} \left(\frac{\nu + H}{\sqrt{2}} \right) \psi_{mR} + .h.c. \right) \quad (43)$$

$$= \sum_{\psi} \bar{\psi}_L (M_{mn}^{\{\psi\}} + h_{mn}^{\{\psi\}} H) \psi_R \quad (44)$$

where the sum over ψ means write one term for each fermion.

$$\psi_L = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}_L \quad (45)$$

$M_{mn}^{\{\psi\}}$ is the fermion mass matrix for the fermion ψ ,

$$M_{mn}^{\{\psi\}} = Y_{mn}^{\{\psi\}} \frac{\nu}{\sqrt{2}} \quad (46)$$

induced by spontaneous symmetry breaking, and

$$h_{mn}^{\{\psi\}} = \frac{1}{\nu} M_{mn}^{\{\psi\}} = \frac{g}{2} \frac{M_{mn}^{\{\psi\}}}{M_W} \quad (47)$$

is the associated Yukawa coupling matrix for the fermion ψ .

In general the fermion mass matrix is not diagonal, Hermitian or symmetric. To identify

the physical particle content it is necessary to diagonalize $M_{mn}^{\{\psi\}}$ by separate unitary transformations \mathcal{A}_L and \mathcal{A}_R on the left and right-handed fermion fields. For example, if the fermion is an up-type quark then,

$$\mathcal{A}_L^{a\dagger} M^b \mathcal{A}_R^c = M_D^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad (48)$$

is a diagonal matrix with eigenvalues equal to the physical masses of the charge $\frac{2}{3}$ quarks. Of course, if M_{mn}^{ψ} is Hermitian one can take $\mathcal{A}_L = \mathcal{A}_R$ without consequence. Similarly, one diagonalizes the down quark and charged lepton mass matrices by

$$\mathcal{A}_L^{d\dagger} M^d \mathcal{A}_R^d = M_D^d \quad (49)$$

$$\mathcal{A}_L^{e\dagger} M^e \mathcal{A}_L^{e\dagger} = M_D^e. \quad (50)$$

In terms of these unitary matrices we can define mass eigenstate fields

$$U_L = \mathcal{A}_L^{u\dagger} U_L^0 = (U_L \ C_L \ T_L)^T \quad (51)$$

$$U_R = \mathcal{A}_R^{u\dagger} U_R^0 = (U_R \ C_R \ T_R)^T \quad (52)$$

with analogous definitions for $D_{L,R} = \mathcal{A}_{L,R}^{d\dagger} D_{L,R}^0$, and $E_{L,R} = \mathcal{A}_{L,R}^{e\dagger} E_{L,R}^0$.

The Lagrangian for the SM after SSB is

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \sum_i \bar{\Psi}_i \left(i \not{\partial}_\mu - m_i - \frac{m_i H}{\nu} \right) \Psi_i \\ & - \frac{g}{2\sqrt{2}} \left(\mathcal{J}_W^\mu W_\mu^- + \mathcal{J}_W^{\mu\dagger} W_\mu^+ \right) - e \mathcal{J}_Q^\mu \mathcal{A}_\mu - \frac{g}{2 \cos \theta_W} \mathcal{J}_Z^\mu \mathcal{Z}_\mu \end{aligned} \quad (53)$$

where the weak charged current is

$$\mathcal{J}_W^{\mu\dagger} = \sum_{m=1}^F [\bar{\nu}_m \gamma^\mu (1 - \gamma^5) E_m + \bar{U}_m \gamma^\mu (1 - \gamma^5) D_m] \quad (54)$$

$$= (\bar{\nu}_e \bar{\nu}_\mu \bar{\nu}_\tau) \gamma^\mu (1 - \gamma^5) \begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} + (\bar{u} \bar{c} \bar{t}) \gamma^\mu (1 - \gamma^5) V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (55)$$

and the quark flavor mixing CKM matrix is

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{td} & V_{td} \end{pmatrix}. \quad (56)$$

The electromagnetic current is given by

$$\mathcal{J}_Q^\mu = \sum_{m=1}^F \left[\frac{2}{3} \bar{u}_m \gamma^\mu u_m - \frac{1}{3} \bar{d}_m \gamma^\mu d_m - \bar{e}_m \gamma^\mu e_m \right] \quad (57)$$

and the Weak Neutral Current is given by

$$\mathcal{J}_Z^\mu = \sum_m [\bar{u}_{mL} \gamma^\mu u_{mL} - \bar{d}_{mL} \gamma^\mu d_{mL} + \bar{\nu}_{mL} \gamma^\mu \nu_{mL} - \bar{e}_{mL} \gamma^\mu e_{mL}] - 2 \sin^2 \theta_W \mathcal{J}_Q^\mu. \quad (58)$$

In the limit $|Q^2| \ll M_W^2$, the momentum term in the W propagator can be neglected, leading to an effective zero-range four Fermi interaction:

$$- \mathcal{L}_{\text{eff}}^{cc} = \frac{G_F}{\sqrt{2}} \mathcal{J}_W^\mu \mathcal{J}_{W\mu}^\dagger \quad (59)$$

where the Fermi constant is identified as

$$\frac{G_F}{\sqrt{2}} \simeq \frac{g^2}{8M_W^2} = \frac{1}{2\nu^2}. \quad (60)$$

Thus, the experimentally measured muon lifetime, $G_F = 1.16639(2) \times 10^{-5} \text{GeV}^{-2}$, implies that the weak interaction scale defined by the vacuum expectation value (VEV) of the Higgs field, is given by:

$$\nu = \sqrt{2} \langle 0 | \varphi | 0 \rangle \simeq 246 \text{ GeV}. \quad (61)$$

1.6 Problems with the SM

The mathematically consistent, renormalizable standard electroweak model, is a field theory which is in agreement with all experimental facts [28]. It successfully predicted the existence and the correct form of the weak neutral current, and the existence and masses of the W and Z bosons. The SM successfully incorporates the generalized Fermi theory with the prediction of the weak charged current interactions. It also successfully incorporates quantum electrodynamics. When combined with quantum chromodynamics and classical general relativity, the predictions of the SM have been verified to a level of precision which imply that it is certainly the approximately correct description of nature down to at least 10^{-18} m. However, the SM is too arbitrary to be truly fundamental, having some 21 free parameters.

There is also the so called gauge problem, which is the fact that there is no explanation for why only the electroweak sector is chiral. Moreover, the standard model incorporates but does not explain charge quantization; it provides no explanation for the reason that all particles have charges which are multiples of $e/3$. Possible explanations include: grand

unified theories (GUTS), the existence of magnetic monopoles, and constraints from the absence or cancellation of anomalies.

Perhaps the most fatal of all problems concerning the SM is the hierarchy problem, which is essentially the question of why the Higgs mass is so light relative to fundamental scales . In the standard model one introduces, by hand, an elementary Higgs field into the theory to generate masses for the W , Z , and fermions. For the model to be consistent we need $m_H^2 = O(m_W^2)$. If m_H is larger than m_W by several orders of magnitude then we arrive at a hierarchy problem. The tree-level, bare, Higgs mass receives quadratically divergent corrections from one loop corrections, i.e. $m_H^2 = (m_H^2)_{\text{bare}} + O(\lambda, g^2, h^2)\Lambda^2$, where Λ is the energy scale where the effects of new physics begins to make non-negligible contributions. We know *ab initio* that the SM breaks down at the scale where gravity becomes strong, i.e the Planck scale, thus the extreme upper limit is $\Lambda = M_P = G_N^{-1/2} \sim 10^{19} \text{ GeV}$.

Naturalness suggests the unification of all forces at some higher energy scale. This occurs in many grand unified theories (GUTs) at scales on the order of $M_X \sim 10^{16} \text{ GeV}$ and thus the natural scale for M_H would be $O(\Lambda)$, much larger than the expected value obtained from the SM. In either case, in order for the SM to make accurate predictions, as it does, there must be a fine tuned, highly contrived cancellation between the bare value and the correction, to more than 30 decimal places in the case of gravity [29]. Moreover, if the cutoff is provided by a grand unified theory, we get an additional hierarchy problem at tree level. The tree level couplings between the Higgs field and the extremely heavy fields lead to the expectation that M_H is equal to the unification scale M_X , unless unnatural fine-tunings are done to an embarrassingly high level of precision.

Baryon B , and Lepton L , number are automatically global symmetries of the SM, thus one may be naturally motivated to consider extensions of the SM in which the individual symmetries of B and L , or combinations there-of are gauged into locally symmetric extensions of the SM gauge group. One reason for doing this, as we will see, is that particular

versions of combined B and L locally symmetric extensions of the SM is their ability to incorporate small non-zero neutrino masses in a natural way, and even posses viable DM candidates without the need going to supersymmetric theories.

The most promising solution to this fine tuning problem is supersymmetry (SUSY) which prevents large renormalization by providing a natural mechanism for cancellations between the various Higgs mass corrections. Thus in order to attack the shortcomings of the SM, as well as the experimental shortcomings presented above, we consider SUSY extensions of the SM as they provide a means to simultaneously resolve these issues.

Chapter 2

Superspace and Supersymmetry

2.1 Introduction and motivation

We have seen that in the SM at tree-level the bare Higgs mass receives quadratically divergent corrections from one loop processes:

$$m_H^2 = (m_H^2)_{\text{bare}} + O(\lambda, g^2, h^2)\Lambda^2. \quad (62)$$

For example, corrections to the Higgs mass appear if the Higgs field couples to a fermion f with a term in the Lagrangian like the following

$$\mathcal{L}_{Hf} = -\lambda_f H \bar{f} f. \quad (63)$$

This term is represented by the Feynman diagram in Figure 1(a), and it yields the following correction to the Higgs mass

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda^2 + \dots \quad (64)$$

Moreover, suppose there exists a heavy complex scalar particle S with mass m_S that couples to the Higgs with a Lagrangian term like $-\lambda_S |H|^2 |S|^2$. The Feynman diagram for this process is shown in Figure 1(b). It gives rise to the following correction to the Higgs mass:

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2} [\Lambda^2 - 2m_S^2 + \ln(\Lambda/m_S) + \dots] \quad (65)$$

Comparing equations (64) and (65) we see that if we consider a new symmetry relating fermions and bosons, then the fine-tuning problems of the SM might be dealt with in a natural way [30]. Motivated along these lines, it is apparent that if we consider supersymmetry, whereby the quarks and leptons of the Standard Model each get accompanied by two

complex scalars with $\lambda_S = 2|\lambda_f|^2$, then

$$(\Delta m_H^2)_f + (\Delta m_H^2)_S = -\frac{\lambda_S}{16\pi^2}\Lambda^2 + \frac{\lambda_S}{16\pi^2}\Lambda^2 - \frac{\lambda_S}{8\pi^2}m_S^2 + \frac{\lambda_S}{16\pi^2}[\ln(\Lambda/m_S) + \dots] \quad (66)$$

$$= -\frac{\lambda_S}{8\pi^2}m_S^2 + \frac{\lambda_S}{16\pi^2}[\ln(\Lambda/m_S) + \dots] \quad (67)$$

evidently the Λ^2 contributions of the loops in Figure 1 (a) and (b) cancel exactly, and we have at most logarithmic divergences which can be summed with renormalization group procedures [31, 32, 33]. A viable theoretical framework that incorporates weakly-coupled Higgs bosons is that of “low energy” or “weak-scale” supersymmetry. In this framework, supersymmetry is used to relate fermion and boson masses and interaction strengths. Supersymmetry is a symmetry whereby every fermionic/bosonic SM particle has an associated superpartner which differs in spin by 1/2.

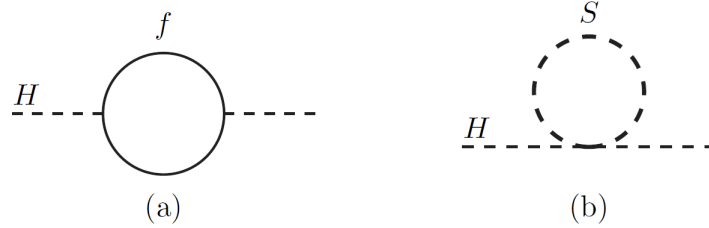


Figure 1: One-loop quantum corrections to the Higgs squared mass parameter m_H^2 , from (a) a Dirac fermion f , and (b) a scalar S .

2.2 The SUSY Algebra and its Representations

A supersymmetry transformation turns a bosonic state into a fermionic state, and vice versa. The operator Q that generates such transformations must be an anti-commuting spinor, with

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle \quad Q|\text{Fermion}\rangle = |\text{Boson}\rangle \quad (68)$$

In a 4-dimensional spacetime the minimal spinor is a Weyl spinor and therefore the minimal supersymmetry has 4 supercharges. Supersymmetry appeared for the first time in 1971, as a

phenomenological consideration. The attempt to explain the neutrino as a Goldstone fermion associated with the spontaneous breaking of a fermionic symmetry compelled Y. Golfand and E. Likhtman to introduce the supersymmetric extension of the Poincare algebra [34].

In 1967 Coleman and Mandula proved a no-go theorem regarding the combination of the newly discovered $SU(3)_i$ internal flavor symmetries with $SU(2)_{spin}$ into $SU(6)$. Coleman and Mandula proved that this combination did not respect S-matrix symmetries [35]. They were only considering the special case where the generators commute. If we also allow for an anti-commuting component, the Coleman Mandula theorem no longer holds.

When we allow for fermionic generators, i.e. anti-commuting generators, the theorem is no longer valid and we can extend the Poincare symmetry of spacetime to include anti-commuting spinorial generators as was shown by Haag, Lopuszański, and Sohnius in 1975 [36]. They proved that supersymmetry is the only additional symmetry of the S-matrix allowed by inclusion of anti-commuting generators of the Poincare algebra. This extension forms a graded-Lie algebra defined by the usual commutation relations of the Poincare symmetry together with the new anti-commutation relations, (where, $\alpha, \beta \in 1, 2$ and $\mu = 0, 1, 2, 3$) :

$$\{Q_\alpha^A, \bar{Q}_\beta^B\} = 2\sigma_{\alpha\beta}^\mu \delta_{AB} P_\mu \quad (69)$$

$$\{Q_\alpha^A, Q_\beta^B\} = \epsilon_{\alpha\beta} X^{AB}; \quad \{\bar{Q}_\alpha^A, \bar{Q}_\beta^B\} = \epsilon_{\dot{\alpha}\dot{\beta}} X^{AB} \quad (70)$$

$$[Q_\alpha^A, P_\mu] = [\bar{Q}_\beta^B, P_\mu] = 0 \quad (71)$$

Here P_μ is the generator of translations and the X^{AB} s are anti-symmetric Lorentz scalars, known as the central charges. The four dimensional, $\mathcal{N} = 1$ SUSY algebra, which forms the basis of the models we consider below, has no central charges since A, B can only take on one value and the X^{AB} then comprise a single anti-commuting Lorentz scalar, which therefore must vanish.

The irreducible massive states in the 4 dimensional $\mathcal{N} = 1$ SUSY theory obey an algebra

which leads to some interesting and important consequences. If we take the trace on both sides of the first equation we obtain

$$Q_1\bar{Q}_1 + \bar{Q}_1Q_1 + Q_2\bar{Q}_2 + \bar{Q}_2Q_2 = 4P^0 \quad (72)$$

This is related to the Hamiltonian

$$\mathcal{H} = P^0 = \frac{1}{4} (Q_1\bar{Q}_1 + \bar{Q}_1Q_1 + Q_2\bar{Q}_2 + \bar{Q}_2Q_2) \quad (73)$$

Any state in the theory that is invariant under a symmetry is annihilated by symmetry generators. In particular, if the ground state $|0\rangle$ is supersymmetric, it will be annihilated by SUSY generators, i.e. $Q_\alpha|0\rangle = 0$.

In general, for a non-zero energy eigenstate we have:

$$\langle E| Q_1\bar{Q}_1 + \bar{Q}_1Q_1 + Q_2\bar{Q}_2 + \bar{Q}_2Q_2|E\rangle = \langle E| 4P^0|E\rangle = 4E \quad (74)$$

Because of the SUSY algebra, non-zero states must come in Fermi-Bose pairs, i.e.

$$Q|B\rangle = \sqrt{E}|F\rangle, \quad Q|F\rangle = \sqrt{E}|B\rangle \quad (75)$$

In global SUSY theories, $E = 0$, and $Q_\alpha|0\rangle = 0$, therefore E is an order parameter for SUSY breaking. The ground state of SUSY is dependent on the order parameter E which parametrizes the ground state of the SUSY theory in its broken $E > 0$, and unbroken $E = 0$ phases. We, therefore, conclude that the energy of a supersymmetric ground state must be zero. On the other hand, if supersymmetry is spontaneously broken, the vacuum energy is positive definite.

2.3 Irreducible Representations of the SUSY Algebra

There are two cases to consider, massless and massive representations. We begin with the *first case*, and seek irreducible representations of massive states, $M > 0$, where $M = \text{mass}$. All massive frames are obtained from Wigner boosts and rotations applied to the rest frame $P_\mu = (M, 0, 0, 0)$ in which the velocity is zero. The SUSY Algebra in this case is an algebra of $2N$ fermion creation $\bar{Q}_{\dot{\beta}B}$, and annihilation Q_α^A operators

$$\{Q_\alpha^A, \bar{Q}_{\dot{\beta}B}\} = 2\delta_{\alpha\dot{\beta}}\delta_B^A P_\mu \quad (76)$$

$$\{Q_\alpha^A, Q_\beta^B\} = \{\bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B}\} = 0 \quad (77)$$

(strictly speaking we must scale each Q by $\frac{1}{\sqrt{2M}}$ to get the standard form of creation and annihilation operators familiar from Quantum Mechanics). An irreducible representation of SUSY in this case has dimension 2^{2N} .

The *second case* is massless, so $M = 0$. The rest frame is given by the four momentum $P_\mu = (E, 0, 0, E)$. Consider a particle moving in the Z direction.

$$\{Q_\alpha^A, \bar{Q}_{\dot{\beta}}^B\} = 2\sigma_{\alpha\dot{\beta}}^\mu \delta_{AB} P_\mu = \delta_{AB} 2E \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_{\alpha\dot{\beta}} \quad (78)$$

For massless case we get N creation and annihilation operators, So the irreducible representation has dimension 2^N and this is an irreducible representation of SUSY. However, if we include the dimensionality of the CPT (equivalent to Lorentz symmetry) generators along with those of SUSY, the general dimension is $2 \cdot 2^N = 2^{N+1}$.

2.4 Superspace and the Superfield Formalism

It would be very useful if we had a way to treat particles and their superpartners as a single field (or superfield). Scalars and fermions related by supersymmetry would simply correspond to different components of a single superfield. To arrive at the desired superfield formalism it is convenient to introduce the notion of superspace by extending the 4 commuting spacetime coordinates x_μ to 4 commuting and 4 anti-commuting coordinates $\{x_\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}\}$, where $\bar{\theta}_{\dot{\alpha}} = (\theta^\alpha)^*$. These co-ordinates satisfy the following anti-commutation relations

$$\{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\} = \{\theta_\alpha, \theta_\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0 \quad (79)$$

We also introduce the integrals over superspace

$$\int d\theta = \int d\bar{\theta} = \int d\theta \bar{\theta} = \int d\bar{\theta} \theta = 0, \quad (80)$$

$$\int d\theta^\alpha \theta_\beta = \delta_\beta^\alpha, \quad \int d\bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = \delta_{\dot{\alpha}}^{\dot{\beta}}, \quad (81)$$

$$\int d^2\theta \theta^2 = \int d^2\bar{\theta} \bar{\theta}^2 = 1, \quad \int d^4\theta \theta^2 \bar{\theta}^2 = 1, \quad (82)$$

where,

$$d^2\theta \equiv -\frac{1}{4}\epsilon_{\alpha\beta}d\theta^\alpha d\theta^\beta, \quad (83)$$

$$d^2\bar{\theta} \equiv -\frac{1}{4}\epsilon_{\dot{\alpha}\dot{\beta}}d\bar{\theta}^{\dot{\alpha}} d\bar{\theta}^{\dot{\beta}}, \quad (84)$$

$$d^4\theta \equiv d^2\bar{\theta} d^2\theta \quad (85)$$

The expansion of functions on superspace coordinates terminates at order $\theta^2\bar{\theta}^2$. This property allows us to express any supermultiplet as a single superfield which depends on super-

space coordinates. The Taylor expansion of the most general scalar superfield is:

$$\Phi(\theta, \bar{\theta}) = \phi + \theta\psi + \bar{\theta}\bar{\psi} + \bar{\theta}\bar{\sigma}^\mu\theta V_\mu + \theta^2 F + \bar{\theta}^2 \bar{F} + \dots + \theta^2 \bar{\theta}^2 D \quad (86)$$

The expansion above is a reducible representation of SUSY. To get irreducible representations we consider chiral Φ , and anti-chiral Φ^\dagger superfields subject to the constraints

$$\bar{D}_{\dot{\alpha}}\Phi = 0, \quad D_\alpha\Phi^\dagger = 0, \quad (87)$$

where,

$$iQ_\alpha = D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu, \quad (88)$$

$$i\bar{Q}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu. \quad (89)$$

If we introduce new variables $y^\mu = x^\mu + i\bar{\theta}\bar{\sigma}^\mu\theta$ and $y^{\mu\dagger} = x^\mu - i\bar{\theta}\bar{\sigma}^\mu\theta$, then

$$\bar{D}_{\dot{\alpha}}y^\mu = D_\alpha y^{\mu\dagger} = 0. \quad (90)$$

Thus we have chiral supermultiplets which are holomorphic functions of θ and defined by

$$\Phi(y^\mu) = \varphi(y^\mu) + \sqrt{2}\theta\psi(y^\mu) + \theta^2 F(y^\mu). \quad (91)$$

In addition, we have real vector supermultiplets which, in the Wess-Zumino gauge, have the following form:

$$V = -\theta\sigma^\mu\bar{\theta}V_\mu + i(\theta^2\bar{\theta}\bar{\lambda} - \bar{\theta}^2\theta\lambda) + \frac{1}{2}\theta^2\bar{\theta}^2 D. \quad (92)$$

The vector supermultiplet contains a vector field V_μ of mass dimension 1, a gaugino λ of mass dimension 3/2, and an auxiliary field D , of mass dimension 2, which will be eliminated by solving the Euler-Lagrange (E-L) equations.

In the following we will consider the MSSM and the $B - L$ extended MSSM. These are both examples of $\mathcal{N} = 1$ supersymmetric theories. Generalizing the results above, the smallest irreducible representations of $\mathcal{N} = 1$ SUSY consists of the following supermultiplets (in the Wess-Zumino gauge):

- A vector superfield (VSF) for each gauge field. The physical particle content of a VSF is one gauge boson and a Weyl fermion called a *gaugino*. The VSF's transform under the adjoint representation of the gauge group.
- A chiral superfield (CSF) for each matter field. The CSF is composed of one spin- $\frac{1}{2}$ Weyl fermion and one spin-0 complex scalar. The CSF's can transform under any representation. Since none of the matter fermions of the SM transform under the adjoint of the gauge group we can not identify them with the gauginos. Thus we have to introduce new fermionic SUSY partners to each SM gauge boson.
- A gravity supermultiplet containing the graviton (spin-2) and its superpartner the gravitino (spin- $\frac{3}{2}$).

2.5 The MSSM

The MSSM is a supersymmetric generalization of the Standard Model gauge group \mathcal{G}_{SM} . It requires a color octet of vector superfields \mathcal{V}^a , an additional weak triplet \mathcal{V}^i , and a hypercharge singlet \mathcal{V} . These superfields contain the appropriate spin-one gauge bosons and their spin- $\frac{1}{2}$ partners as displayed in table 3. In the MSSM the vector superfield generalizations of the SM gauge fields, interact with the superfield generalization of the quarks and leptons. These superfields are also shown in Table 3. They are chiral superfields; they contain the spin- $\frac{1}{2}$ quarks and leptons, as well as their spin zero partners, the squarks and sleptons.

The supersymmetric extensions of Higgs bosons are also shown in Table 3. They include two complex Higgs doublets (H_u, H_d) , as well as their spin- $\frac{1}{2}$ partners, the two Higgsinos. We introduce the extra fields because in SUSY theories, two (or more) Higgs doublets are required for the Higgsino anomalies to cancel among themselves, and they are also required to reproduce all the SM Yukawa couplings.

CSF	$SU(3)$	$SU(2)$	$U(1)$	B	L	Particles
L_i	1	2	$-\frac{1}{2}$	0	1	leptons (ν, e) and sleptons $(\tilde{\nu}, \tilde{e})$
E_i^c	1	1	1	0	-1	electron e^c and selectron \tilde{e}^c
Q_i	3	2	$+\frac{1}{6}$	$\frac{1}{3}$	0	quarks (u, d) and squarks (\tilde{u}, \tilde{d})
U_i^c	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	quarks u^c and squarks \tilde{u}^c
D_i^c	$\bar{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	0	quarks d^c and squarks \tilde{d}^c
H_u	1	2	$\frac{1}{2}$	0	0	Higgs h_u and Higgsinos \tilde{h}_u
H_d	1	2	$-\frac{1}{2}$	0	0	Higgs h_d and Higgsinos \tilde{h}_d
VSF	$SU(3)$	$SU(2)$	$U(1)$	B	L	Particles
\mathcal{V}^a	8	1	0	0	0	gluons g and gluinos \tilde{g}
\mathcal{V}^i	1	3	0	0	0	W 's and winos \tilde{W}
\mathcal{V}	1	1	0	0	0	B and bino \tilde{B}

Table 3: The Particle contents of the MSSM ($i = 1, 2, 3$)

Once the particle content is fixed one can try to write down the most general renormalizable Lagrangian for this $\mathcal{N} = 1$ supersymmetric $SU(3) \otimes SU(2) \otimes U(1)$ theory. It is well known from the structure of $\mathcal{N} = 1$ SUSY gauge theories that the Lagrangian is completely fixed by gauge invariance and by supersymmetry, except for the choice of the superpotential, whose most general form contains all possible gauge invariant operators of dimension 3 or less. The following superpotential contains all such operators.

$$\mathcal{W} = Y_u^{ij} Q^i H_u \bar{U}^j + Y_d^{ij} Q^i H_d \bar{D}^j + Y_e^{ij} L^i H_d \bar{E}^j + \mu H_d H_u + \mathcal{W}_{\mathbb{R}_p} \quad (93)$$

where the baryon and lepton number violating contributions are contained in:

$$\mathcal{W}_{\mathbb{R}_p} = a_1^{ijk} Q^i L^j \bar{D}^k + a_2^{ijk} L^i L^j \bar{E}^k + a_3^i L^i H_u + a_4^{ijk} \bar{D}^i \bar{D}^j \bar{U}^k \quad (94)$$

The terms on the first line, with the exception of $\mathcal{W}_{\mathbb{R}_p}$, correspond to the SUSY generalization of the ordinary Yukawa interactions of the SM plus an additional μ -term breaking the Peccei-Quinn degeneracy of the two Higgs doublet model. The terms on the second line are generally allowed since they are gauge invariant, but lead to lepton and baryon number violating interactions. Baryon and lepton number non-conservation is at odds with the situation in the SM where the most general renormalizable gauge invariant Lagrangian automatically conserved baryon and lepton number. To proceed we must impose additional symmetries which forbid B and L violating interactions that are dangerous phenomenologically.

The easiest way to achieve this is to introduce R-parity and require that it be conserved. The R-parity of a field is given by

$$R = (-1)^{3B+L+2S} \quad (95)$$

where B is the baryon number, L the lepton number, and S the spin of a given particle associated with a particular field. One could also forbid the appearance of the B and L breaking terms by imposing other, different symmetry requirements. For example a Z_2 subgroup of $B \otimes L$ known as matter parity,

$$P = (-1)^{3(B-L)} \quad (96)$$

could achieve this goal as well. Actually matter parity and R-Parity are equivalent for any

vertex that conserves angular momentum [37]. The important point is that once lepton and baryon number violating terms are absent, R-parity will necessarily be a symmetry of the Lagrangian regardless of its ultimate status as primary or accidental.

Another strong motivation for the introduction of R-Parity is that it provides a dark matter candidate, namely the Lightest Supersymmetric Particle (LSP). One consequence of R-parity conservation is that super-partners may only be produced in pairs, implying that the LSP is stable if R-parity is exactly conserved.

2.6 Supersymmetry Breaking

If SUSY were exact then the masses of all the sparticles would be identical to their SM counterparts. For example, there would be a particle, the selectron, which had the same mass as the electron, and we would certainly have seen this particle by now. Therefore SUSY, if it exists, must exist as a broken symmetry in nature. There are two possibilities for SUSY breaking, explicit SUSY breaking and spontaneous SUSY breaking. A model whose Lagrangian density is not invariant under supersymmetric transformations explicitly breaks SUSY. On the other hand, a spontaneous SUSY breaking model is one whose Lagrangian density is invariant under supersymmetry, but whose vacuum state is not. Spontaneous SUSY breaking is favored from a theoretical point of view because we would always prefer our Lagrangian densities to be SUSY invariant in a SUSY theory, however this is not a viable option in the MSSM [38].

Our only remaining option then is to introduce explicit SUSY breaking terms in order to break SUSY. However these terms should not come at the price of sacrificing the solution to the hierarchy problem. Such terms are called soft SUSY breaking terms, and those are the terms that do not reintroduce quadratic divergences into the theory. The soft supersymmetry breaking Lagrangian is defined to include all allowed terms that do not introduce quadratic

divergences in the theory: all gauge invariant and Lorentz invariant terms of dimension two and three. The complete set of possible soft SUSY breaking parameters was elucidated by K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita [39], and the classic proof provided by Girardello and Grisaru [40]. The \mathcal{L}_{soft} terms are of the following types, where the summation convention is implied throughout:

- Soft tri-linear scalar interactions:

$$\frac{1}{3!} \mathcal{A}_{ijk} \phi_i \phi_j \phi_k + h.c. \quad (97)$$

- Soft bi-linear scalar interactions:

$$\frac{1}{2} \mathcal{B}_{ij} \phi_i \phi_j + h.c. \quad (98)$$

- Soft scalar mass-squares:

$$m_{ij}^2 \phi_i^\dagger \phi_j \quad (99)$$

- Soft gaugino masses (where, a is a group label):

$$\frac{1}{2} \mathcal{M}_a \lambda^a \lambda^a + h.c. \quad (100)$$

The basic idea behind the soft breaking terms is that there exists a sector of physics that breaks SUSY spontaneously. This sector resides at energy scales much higher than the weak scale. SUSY breaking is then communicated in some way (either through gauge interactions or through gravity) to the MSSM fields and as a result the soft breaking terms appear.

A common implementation proceeds to break SUSY spontaneously in a hidden sector which is decoupled from the SM particles in the visible sector, except through supergravity which will mediate the SUSY breaking terms to the visible sector. The minimal supergravity mediation mechanism generates universal soft breaking terms for the visible sector fields at the Planck scale. Thus one has to think of the MSSM as an effective theory, valid below a certain scale (of new physics), and the soft breaking terms will parametrize our ignorance of the details of the physics of the SUSY breaking sector.

2.7 mSUGRA and the CMSSM

The most general soft SUSY breaking terms introduce 105 new parameters into our model, this is definitely not a good thing because of known constraints on FCNC's and CP violation. The most minimal approach to mediating supersymmetry breaking between hidden and visible sectors is through supergravity interactions. Generically one should expect that the most general interactions consistent with the symmetries of both hidden and visible sector will be generated in an effective theory with Planck suppressed couplings. The Gravity mediation of SUSY breaking from the hidden sector is flavor blind and so it drastically reduces the number of free parameters by justifying the assumption of diagonal SUSY breaking mass matrices. In the mSUGRA mediation scenario, we obtain the so-called constrained MSSM or CMSSM. In this case we assume unification of the following parameters at the GUT scale \mathcal{M}_{GUT} [41]:

- Universal gaugino masses (at the the GUT Scale):

$$\mathcal{M}_3 = \mathcal{M}_2 = \mathcal{M}_1 = \mathcal{M}_{1/2} \quad (101)$$

- Universal scalar masses(sfermions and Higgs) masses(at the the GUT Scale):

$$\mathcal{M}_Q^2 = \mathcal{M}_L^2 = \mathcal{M}_{U^c}^2 = \mathcal{M}_{D^c}^2 = \mathcal{M}_{E^c}^2 = \mathcal{M}_{H_u}^2 = \mathcal{M}_{H_d}^2 = m_0^2 \quad (102)$$

- Universal tri-linear couplings (at the the GUT Scale):

$$A_u = A_d = A_L = A_0 \quad (103)$$

This reduces the number of free parameters from 124 to just 5 [42]:

$$\tan \beta, \quad M_{1/2}, \quad m_0, \quad A_0 \quad \text{sign}(\mu) \quad (104)$$

Where, $\tan \beta = |\langle H_u \rangle| / |\langle H_d \rangle|$, and μ is the co-efficient of the Peccei-Quinn term in \mathcal{W}_{MSSM} .

2.8 The MSSM Higgs Sector

The Higgs scalar potential is given by

$$V = V_D + V_F + V_{soft} \quad (105)$$

The term V_D represents the D-term potential which is obtained from

$$V_D = \sum_A \frac{1}{2} D^A D^A \quad (106)$$

where

$$D^A \equiv -g_A \phi_i^* T_{ij}^A \phi_j \quad (107)$$

The $U(1)_Y$ contribution to D term is

$$D^1 = -\frac{g'}{2} (|H_u|^2 - |H_d|^2) \quad (108)$$

and the $SU(2)$ contribution to the D term is (where $T^a = \frac{\tau^a}{2}$):

$$D^a = -\frac{g}{2} (H_d^{i*} \tau_{ij}^a H_d^j + H_u^{i*} \tau_{ij}^a H_u^j)$$

Thus the D-terms contribute the following to the scalar potential

$$V_D = \frac{g'^2}{8} (|H_d|^2 - |H_u|^2)^2 + \frac{g^2}{8} (H_d^{i*} \tau_{ij}^a H_d^j + H_u^{i*} \tau_{ij}^a H_u^j)^2 \quad (109)$$

Using the $SU(2)$ identity

$$\tau_{ij}^a \tau_{kl}^a = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} \quad (110)$$

we may write this as:

$$V_D = \frac{g^2}{8} [4|H_u^* \cdot H_d|^2 - 2(H_u^* \cdot H_d)(H_d^* \cdot H_d) + (|H_u|^2 + |H_d|^2)] + \frac{g'^2}{8} (|H_d|^2 - |H_u|^2)^2 \quad (111)$$

which, upon recalling the definitions

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \quad (112)$$

can be written in component form as:

$$V_D(H_u, H_d) = \frac{g^2 + g'^2}{8} (|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \frac{g^2}{2} |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 \quad (113)$$

The F -term is given by $V_F = \sum_i |F_i|^2$, where $F_i^* = \frac{\partial \mathcal{W}}{\partial \phi_i}$, and the ϕ_i are the scalar components of the chiral superfields in the MSSM superpotential. The F -term component of the scalar potential is :

$$V_F = \mu^2 (|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2) \quad (114)$$

The soft SUSY breaking terms in the potential are given by (\tilde{m}_u^2 and \tilde{m}_d^2 are soft Higgs masses):

$$\begin{aligned} V_{soft} = & \tilde{m}_u^2 (|H_u^0|^2 + |H_u^+|^2) + \tilde{m}_d^2 (|H_d^0|^2 + |H_d^-|^2) + \\ & + B (H_u^+ H_d^- - H_u^0 H_d^0) + B (H_u^{+*} H_d^{-*} - H_u^{0*} H_d^{0*}) \end{aligned} \quad (115)$$

EWSB requires that the Higgs potential is bounded from below and has a minimum at non-vanishing VEVs. The Higgs mass matrix will satisfy these conditions if

$$|B|^2 > (\tilde{m}_u^2 + |\mu|^2)(\tilde{m}_d^2 + |\mu|^2) \quad (116)$$

and,

$$2\mu^2 + \tilde{m}_u^2 + \tilde{m}_d^2 > 2|B|. \quad (117)$$

2.9 The Renormalization Group Equations

The MSSM is free of quadratic divergences, RGE evolution modifies relations between superpartner masses and may lead to radiative EWSB. The dominant effect arises from the Higgs interactions with the third generation where (where $t = \frac{1}{16\pi^2} \log \frac{M_{GUT}^2}{\Lambda^2}$) :

$$\frac{d}{dt} \begin{pmatrix} \tilde{m}_u^2 \\ \tilde{m}_t^2 \\ \tilde{m}_{Q_3}^2 \end{pmatrix} = Y_t \begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} - A_t^2 \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad (118)$$

We can see that H_u receives the largest negative contribution and once its mass is driven negative electroweak symmetry is broken. Thus we can see that the radiative corrections due to the top Yukawa coupling want to reverse the sign of the soft breaking mass parameter of the up-type Higgs, which is enough to satisfy the conditions for electroweak breaking at the weak scale. Appropriate choices of the input parameters $M_{1/2}$, m_0 , A_0 and λ_t will drive the soft breaking mass parameter of the up-type Higgs negative which will result in the breaking of electroweak symmetry. This mechanism is called radiative electroweak symmetry breaking.

By requiring that the parameters in the Higgs potential lead to experimentally observed Z and W mass we obtain relations between soft parameters which must be satisfied at the weak scale:

$$\mu^2 = \frac{\tilde{m}_u^2 - \tilde{m}_d^2 \tan \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2 \quad (119)$$

$$B = \frac{1}{2} (\tilde{m}_u^2 + \tilde{m}_d^2 + 2\mu^2) \sin 2\beta \quad (120)$$

where $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$.

From this expression we see that naturalness requires that both μ and B are electroweak scale parameters. However, the μ -term is a supersymmetric term in the Lagrangian and could

take any value between EWSB and Planck scales. This leads to the so-called μ -problem. Only models where the μ -term arises as a result of SUSY breaking are expected to avoid fine-tuning. Even then, the absence of fine-tuning is not guaranteed.

The MSSM solves the hierarchy problem of the SM but it still cannot explain the origin of neutrino masses. The B-L extension of the MSSM allows for a natural implementation of the see-saw mechanism, the well known natural way to obtain small non-zero neutrino masses.

Chapter 3

The B-L Extension of The MSSM

3.1 Introduction

It is well-known that the relevant scale for neutrino mass generation through the seesaw mechanism is given by the $B - L$ scale. The global $B-L$ symmetry present in the SM is an accidental anomaly free symmetry, and once it is gauged, the $U(1)_{B-L}^3$ and $U(1)_{B-L}$ anomalies must be canceled. The $B-L$ anomaly cancellation conditions can be satisfied by introducing three generations of right-handed neutrinos, which are singlets under the SM.

The particle content of the Supersymmetric $B - L$ extended Standard Model includes the following fields in addition to those of the MSSM:

- Three chiral right-handed neutrino superfields $\{\nu_1^c, \nu_2^c, \nu_3^c\}$.
- The Z_{B-L} vector superfield necessary to gauge the $U(1)_{B-L}$ symmetry.
- Two chiral standard model singlet Higgs superfields (X, \bar{X}) with $B - L$ charges $(-2, 2)$ respectively.

As is the case in the MSSM, the introduction of a second Higgs singlet \bar{X} is necessary in order to cancel the $U(1)_{B-L}$ anomalies produced by the fermionic member of the first Higgs superfield X . The charges for quark and lepton superfields are assigned in the usual way. We present the particle contents of all supermultiplets of the $B - L$ extended MSSM below in table 4:

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
Q_i	3	2	$+\frac{1}{6}$	$+\frac{1}{3}$
U_i^c	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{1}{3}$
D_i^c	$\bar{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$
L_i	1	2	$-\frac{1}{2}$	-1
E_i^c	1	1	1	+1
ν_1^c	1	1	0	+1
ν_2^c	1	1	0	+1
ν_3^c	1	1	0	+1
H_u	1	2	$+\frac{1}{2}$	0
H_d	1	2	$-\frac{1}{2}$	0
X	1	1	0	-2
\bar{X}	1	1	0	+2

Table 4: Particle Contents of $B - L$ extended MSSM

The superpotential for the model is [43]:

$$\mathcal{W} = \mathcal{W}_{MSSM} + \mathcal{W}_{B-L} \quad (121)$$

$$\mathcal{W}_{MSSM} = Y_u Q H_u u^c + Y_d Q H_d d^c + Y_e L H_d e^c + \mu H_u H_d \quad (122)$$

$$\mathcal{W}_{B-L} = Y_\nu L H_u \nu^c + f \nu^c \nu^c X - \mu_X X \bar{X} \quad (123)$$

The soft supersymmetric breaking Lagrangian is

$$-\mathcal{L}_{soft} = a_\nu L H_u \nu^c - a_X \nu^c \nu^c X - b_X X \bar{X} + \frac{1}{2} M_{BL} B' B' + h.c. \quad (124)$$

$$+ m_X^2 X^2 + m_{\bar{X}}^2 \bar{X}^2 + m_{\nu^c}^2 (\nu^c)^2 \quad (125)$$

Where B' is the $B - L$ gaugino. Spontaneous $B - L$ violation requires either the VEV of X , \bar{X} or ν^c to be nonzero, however R-Parity conservation depends on the VEV of ν^c :

$\langle \nu^c \rangle = 0$ corresponds to R -parity conservation while $\langle \nu^c \rangle \neq 0$ yields spontaneous R -parity violation [43].

In order to investigate the values of these VEV's we need the minimization conditions which are derived from the full potential, (we use the parametrization: $(\langle X \rangle, \langle \bar{X} \rangle, \langle \nu^c \rangle) = 1/\sqrt{2}(x, \bar{x}, n)$), which is:

$$\langle V \rangle = \langle V_f \rangle + \langle V_D \rangle + \langle V_{soft} \rangle \quad (126)$$

where,

$$\langle V_f \rangle = \frac{1}{4}f^2n^4 + f^2n^2x^2 + \frac{1}{2}\mu_X(x^2 + \bar{x}^2) - \frac{1}{\sqrt{2}}f\mu_xn^2\bar{x} \quad (127)$$

$$\langle V_D \rangle = \frac{1}{32}g_{BL}^2(2\bar{x}^2 - 2x^2 + n^2)^2 \quad (128)$$

$$\langle V_{soft} \rangle = -\frac{1}{\sqrt{2}}a_Xn^2x - b_Xx\bar{x} + \frac{1}{2}m_X^2x^2 + \frac{1}{2}m_{\bar{X}}^2\bar{x}^2 + \frac{1}{2}m_{\nu^c}n^2 \quad (129)$$

There are two mechanisms capable of inducing spontaneous $B - L$ violation, which are what we, and the authors in [43] have classified as:

- Case 1: $(n = 0; x \neq 0, \bar{x} \neq 0)$ is the R -parity conserving case.
- Case 2: $(x \neq 0, \bar{x} \neq 0, n \neq 0)$ is the R -parity violating case.

There exists a third case:

- Case 3: $(n \neq 0; x = 0, \bar{x} = 0)$,

which cannot exist due to the linear term for x in V_{soft} , and the linear term for \bar{x} in V_F which always induce a VEV for these fields.

3.2 The R-Parity Conserving Case

The minimization conditions for x and \bar{x} are very similar in form to those of v_u and v_d in the MSSM

$$\frac{1}{2}M_{Z'}^2 = -|\mu_X|^2 + \frac{m_X^2 \tan^2 z - m_{\bar{X}}^2}{1 - \tan^2 z} \quad (130)$$

where $\tan z \equiv x/\bar{x}$ and $M_{Z'}^2 \equiv g_{BL}^2 (x^2 + \bar{x}^2)$, which is the mass for the Z' boson associated with broken $B - L$. In this case it is useful to examine the limit where $x \gg \bar{x}$, with $m_X^2 < 0$ and $m_{\bar{X}}^2 > 0$, in which equation 130 reduces to:

$$\frac{1}{2}M_{Z'}^2 = -|\mu_X|^2 - m_X^2 \quad (131)$$

so that,

$$g_{BL}^2 (x^2 + \bar{x}^2) = -2 (|\mu_X|^2 + m_X^2) \quad (132)$$

We see that for $B - L$ breaking to occur we require, in addition to $m_X^2 < 0$, that the magnitudes obey the constraint $-m_X^2 > |\mu_X|^2$.

This model holds promise for explaining neutrino masses. Replacing X by its VEV in the term $f\nu^c\nu^c X$ in the superpotential leads to the heavy Majorana mass term for the right-handed neutrinos and ultimately to the Type I seesaw mechanism for neutrino masses:

$$m_\nu = v_u^2 Y_\nu^T (f x)^{-1} Y_\nu \quad (133)$$

As can be seen from equation 132, x is of order the SUSY mass scale or about a TeV. Moreover, given that the mass of the right-handed neutrinos are of order TeV, scenarios which yield realistic neutrino masses require, $Y_\nu \sim 10^{-6-7}$.

3.3 The R-Parity Violating Case

The evaluation of the minimization conditions in this case is illuminating in the limit where $g_{BL}^2 \ll 1$, and $n \gg \{x, \bar{x}, a_X\}$:

$$n^2 = \frac{-m_{\nu^c}^2 \Lambda_{\bar{X}}^2}{f^2 m_X^2 + \frac{1}{8} g_{BL}^2 \Lambda_{\bar{X}}^2} \quad (134)$$

$$\bar{x} = \frac{(-m_{\nu^c}^2) f \mu_X}{\sqrt{2} (f^2 m_X^2 + \frac{1}{8} g_{BL}^2 \Lambda_{\bar{X}}^2)} \quad (135)$$

$$x = \frac{(-m_{\nu^c}^2) [a_X \Lambda_{\bar{X}}^2 + f b_X \mu_X]}{(2 f^2 - \frac{1}{4} g_{BL}^2) (-m_{\nu^c}^2) \Lambda_{\bar{X}}^2 + f^2 m_X^2 \Lambda_{\bar{X}}^2 + \frac{1}{8} g_{BL}^2 \Lambda_{\bar{X}}^2 \Lambda_X^2} \quad (136)$$

where, $\Lambda_X^2 \equiv \mu_X^2 + m_X^2$ and $\Lambda_{\bar{X}}^2 \equiv \mu_X^2 + m_{\bar{X}}^2$.

The Z' mass in the R -parity violating case is given by

$$M_{Z'}^2 = \frac{1}{4} (n^2 + 4x^2 + 4\bar{x}^2) \quad (137)$$

We can see from these equations that spontaneous $B - L$ symmetry breaking in the R -parity violating case only requires $m_{\nu^c}^2 < 0$. Moreover, there is no introduction of a new μ problem so that μ_X can be larger than the TeV scale. The $\mu \rightarrow \infty$ serves as a decoupling limit since $x, \bar{x} \rightarrow 0$ and $n^2 \rightarrow -8m_{\nu^c}/g_{BL}^2$ as in the minimal model.

3.4 The RGE's and RSBM analysis

We will investigate $B - L$ parity conservation/violation in these different parameter subspaces by evolving the RGEs down from the GUT scale to the TeV scale. We use the gravitational mediation of SUSY breaking in our analysis and here we will adopt the

mSUGRA ansatz with the following boundary conditions at the GUT scale:

$$m_X^2 = m_{\bar{X}}^2 = m_{\nu_i^c}^2 = m_{MSSM}^2 = m_0^2 \quad (138)$$

The boudary mass term m_0 is the universal scalar mass at the GUT scale, and m_{MSSM}^2 indicates the relevant set of MSSM parameters. The trilinear couplings also unify at the GUT scale, as do the gauginos as shown below. In this case Y_{MSSM} is the universal Yukawa coupling at the GUT scale.

$$A_X = f A_0; \quad A_\nu = Y_\nu A_0; \quad A_{MSSM} = Y_{MSSM} A_0 \quad (139)$$

$$M_{BL} = M_{MSSM} = M_{1/2} \quad (140)$$

Where $M_{1/2}$ is the universal gaugino mass.

We utilize the following renormalization group equations (RGEs), subjecting them to the boundary conditions listed above.

The RGEs are given by [44] (assuming a flavor diagonal basis $f_1 = f_2 = f_3$) where $i = 1, 2, 3$:

$$16\pi^2 \frac{dg_{BL}}{dt} = 9 g_{BL}^3 \quad (141)$$

$$16\pi^2 \frac{dM_{BL}}{dt} = 18 g_{BL}^2 M_{BL} \quad (142)$$

$$16\pi^2 \frac{df_i}{dt} = f_3 \left(8 f_i^2 + 2 \text{Tr } f^2 - \frac{9}{2} g_{BL}^2 \right) \quad (143)$$

$$16\pi^2 \frac{da_{X_i}}{dt} = f_X \left(16 f_i a_{X_i} + 4 \text{Tr } (f a_X) - 9 g_{BL}^2 M_{BL} \right) + a_{X_i} \left(8 f_i^2 + 2 \text{Tr } f^2 - \frac{9}{2} g_{BL}^2 \right) \quad (144)$$

The Running of the soft-masses is given by (for $i = 1, 2, 3$):

$$16\pi^2 \frac{dm_X^2}{dt} = -12 g_{BL}^2 M_{BL}^2 \quad (145)$$

$$16\pi^2 \frac{dm_X^2}{dt} = 4 \text{Tr} f^2 m_X^2 + 8 \text{Tr} (f^2 m_{\nu^c}^2) + 4 \text{Tr} a_X^2 - 12 g_{BL}^2 M_{BL}^2 \quad (146)$$

$$16\pi^2 \frac{dm_{\nu_i^c}^2}{dt} = 8 f_i^2 (m_X^2 + 2 m_{\nu_i^c}^2) + 8 a_{X_i}^2 - 3 g_{BL}^2 M_{BL}^2 \quad (147)$$

Radiative symmetry breaking requires one of the soft masses to run negative.

3.5 Results

We have selected representative solutions for the different solution subspaces. We chose specific values for the GUT scale couplings below, however there is nothing inherently special about these numbers *per se*. We simply pick them to illustrate the observation of the existence of three different solution subspaces. One could change the boundary conditions to obtain different results for each solution space in order to examine the details of any particular situation. Our goal below is to present a single example of each of the three possible scenarios, with boundary conditions characteristic of the solution subspace in question.

- (Case 1) For the R -parity conserving case, we have the following boundary conditions at the GUT scale $M_{GUT} = 3 \cdot 10^{16}$ GeV. We let $\mu_X = 2000$ GeV in all cases.

$$f_1 = 0.855, f_2 = 0.935, f_3 = 1.095, g_{BL}^2 = 0.53, M_{1/2} = 500, m_0 = 5000, A_0 = 0 \quad (148)$$

We let the tri-linear couplings A_0 vanish as they have very little effect on the overall running of the parameters.

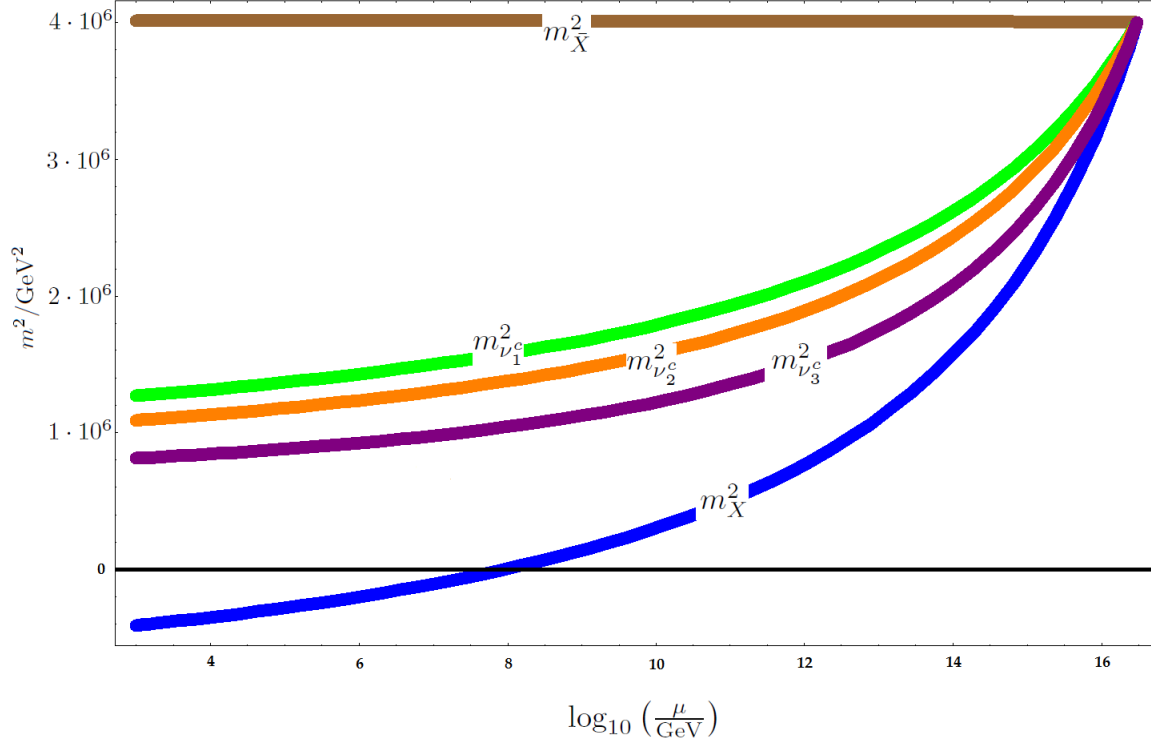


Figure 2: RGE running of the scalar masses in R -Parity conserving case (case 1) of the $B - L$ model.

Examining figure 2, we see that the mass-squared of the $B - L = -2$ Higgs becomes negative, breaking the $B - L$ symmetry but preserving R -parity. In this case the neutralino, the MSSM LSP, is still a viable dark matter candidate since it is forbidden to decay through R -parity conserving channels.

- (Case 2) For the R -Parity Violating Case, we have the following boundary conditions at the GUT scale $M_{GUT} = 3 \cdot 10^{16}$ GeV.

$$f_1 = f_2 = 0.4, f_3 = 3, g_{BL}^2 = 0.53, M_{1/2} = 500, m_0 = 2000, A_0 = 0 \quad (149)$$

We let the tri-linear couplings A_0 vanish, as they have very little effect on the overall running of the parameters.

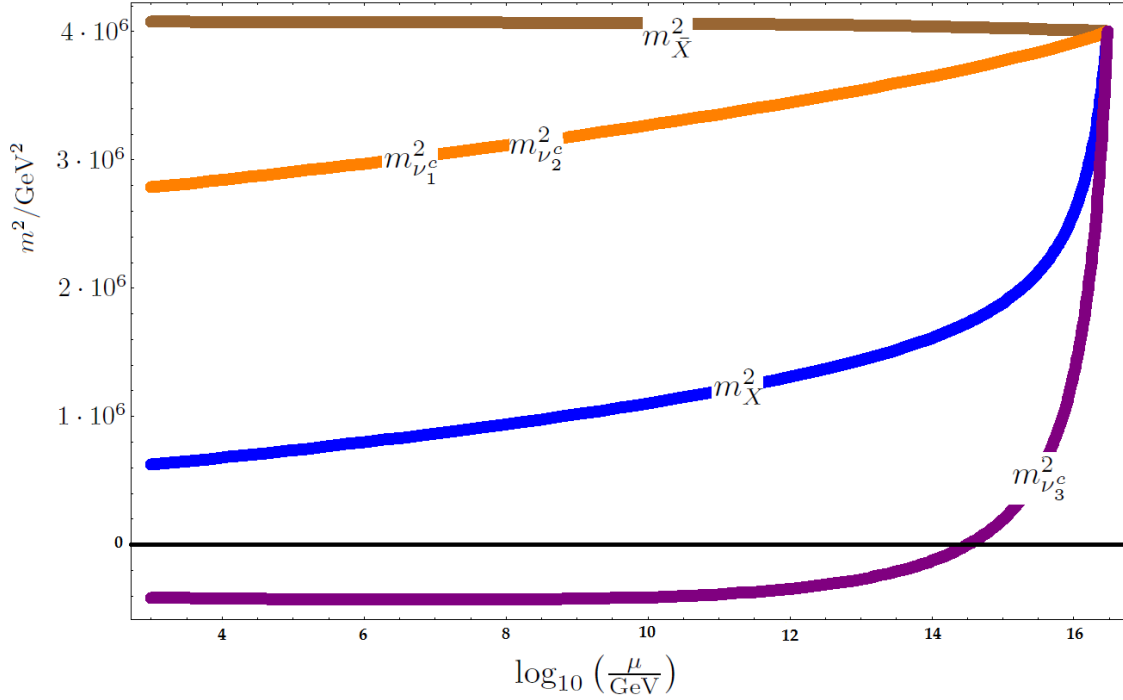


Figure 3: RGE running of the scalar masses in R -Parity violating case (case 2) of the $B - L$ model.

Examining figure 3, we see that in this case the 3rd ($i = 3$) right handed neutrino, ν_3^c runs negative breaking $B - L$, and simultaneously violating R -parity.

- (Case 3) This case does not violate R -parity or $B - L$. To obtain a representative solution of this type let us take the following boundary conditions at the GUT scale $M_{GUT} = 3 \cdot 10^{16}$ GeV.

$$f_1 = f_2 = 1.25, f_3 = 0.01, g_{BL}^2 = 0.53, M_{1/2} = 500, m_0 = 2000, A_0 = 0 \quad (150)$$

We let the tri-linear couplings A_0 vanish as they have very little effect on the overall running of the parameters. The situation with large Yukawa couplings for the first and second right handed neutrinos relative to the third leaves $B - L$ unbroken.

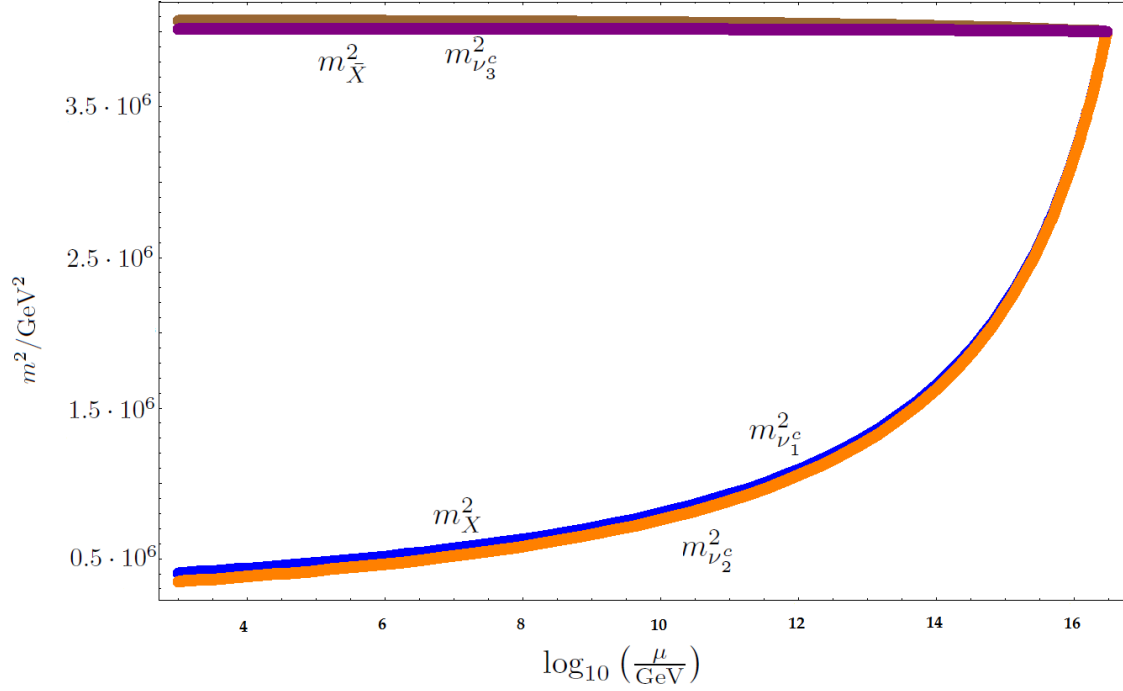


Figure 4: RGE running of the scalar masses in the dormant case (case 3) of the $B - L$ model.

As can be seen from figure 4, with these boundary conditions none of the mass-squareds become tachyonic, producing a phenomenologically dormant situation, never the less we include it here for completeness.

3.6 Additional \mathbb{Z}_2 parity and DM candidates in R -parity violating case.

We have examined the $B - L$ extended MSSM and found that there is a region of parameter space associated with this model that violates R -parity even when we utilize mSUGRA mediation which protects the MSSM. To this end we postulate a new \mathbb{Z}_2 parity under which ν_1 is \mathbb{Z}_2 odd, and everything else is \mathbb{Z}_2 even. Thus ν_1 becomes a potential dark matter candidate, in addition to the gravitino; we display the particle contents below with this additional \mathbb{Z}_2 parity.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	Z_2	$U(1)_{B-L}$
Q_i	3	2	$+\frac{1}{6}$	+	$+\frac{1}{3}$
U_i^c	$\bar{3}$	1	$-\frac{2}{3}$	+	$-\frac{1}{3}$
D_i^c	$\bar{3}$	1	$\frac{1}{3}$	+	$-\frac{1}{3}$
L_i	1	2	$-\frac{1}{2}$	+	-1
E_i^c	1	1	1	+	+1
ν_1^c	1	1	0	-	+1
ν_2^c	1	1	0	+	+1
ν_3^c	1	1	0	+	+1
H_u	1	2	$+\frac{1}{2}$	+	0
H_d	1	2	$-\frac{1}{2}$	+	0
X	1	1	0	+	-2
\bar{X}	1	1	0	+	+2

We plan to investigate the phenomenology of this case in a forthcoming paper.

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